Markov Chain Exercises

SDMC - Gauss

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1. Which of these processes are Markov processes and which are not? For each part, if the problem describes a Markov process, draw a state diagram, write the transition matrix, and determine whether the process is regular. Otherwise, say why the process is not Markov.
   1. Math, Inc. is a large company that employs mathematicians. 5% of mathematicians not employed by Math, Inc. will join the company each year. 98% of the mathematicians employed by Math, Inc. who have worked there at least five years will choose to stay for another year, while the remaining 2% will quit. Only 85% of the newer employees (those who have worked at Math, Inc. for less than five years) will choose each year to remain with the company.
   2. Sometimes you bike to school, and other times you walk. You prefer to vary how you get to school, so if you used one method of transportation yesterday, there is a 10% chance that you will use the same method today. Otherwise, you will switch.
   3. A particle moves on each of the eight vertices of a cube, where it is equally likely at each step to move to each of the adjacent vertices.
   4. You choose to move from one place to another by a series of jumps and somersaults. Your first move is a jump. After each jump, there is a 75% chance you will jump again, and a 25% chance you will switch to doing somersaults. You like to do somersaults, but if you do too many, you get dizzy. If you’ve already done 3 somersaults in a row, you switch to jumping with probability 1. If you’ve done less than 3 somersaults, then your next move will be a somersault with probability 0.06. That is, with probability 0.4, you switch back to jumping.
2. We used the fact that xk=Pkx0. Prove formally that this is true for all integer values of k.
3. One way of finding the steady-state vector is to find the eigenvectors of the transition matrix. Suppose an n-by-n transition matrix has eigenvectors v1, v2, … vn corresponding to distinct eigenvalues λ1, λ2, … λn. To find the steady-state vector, we need to find the inverse of the matrix of whose columns are v1, v2, … vn. Why is this matrix always invertible?
4. Snakes and Ladders is one game that can be modeled by a Markov process. Think of another game that can be modeled by a Markov process and use what you learned to figure out the number of turns after which you can expect, with 95% probability, to complete the game.
5. Suppose the internet consisted of six web pages linked together in the following way:

page 1 links to page 2

page 1 links to page 6

page 2 links to page 1

page 2 links to page 6

page 3 links to page 1

page 3 links to page 5

page 5 links to page 1

page 5 links to page 3

page 5 links to page 6

Suppose that Google’s Page Rank Algorithm assumes that 88% of the time, a random surfer will click a link on the current page, and 12% of the time, they will type in a URL to get to any other page. How would the algorithm rank these pages?